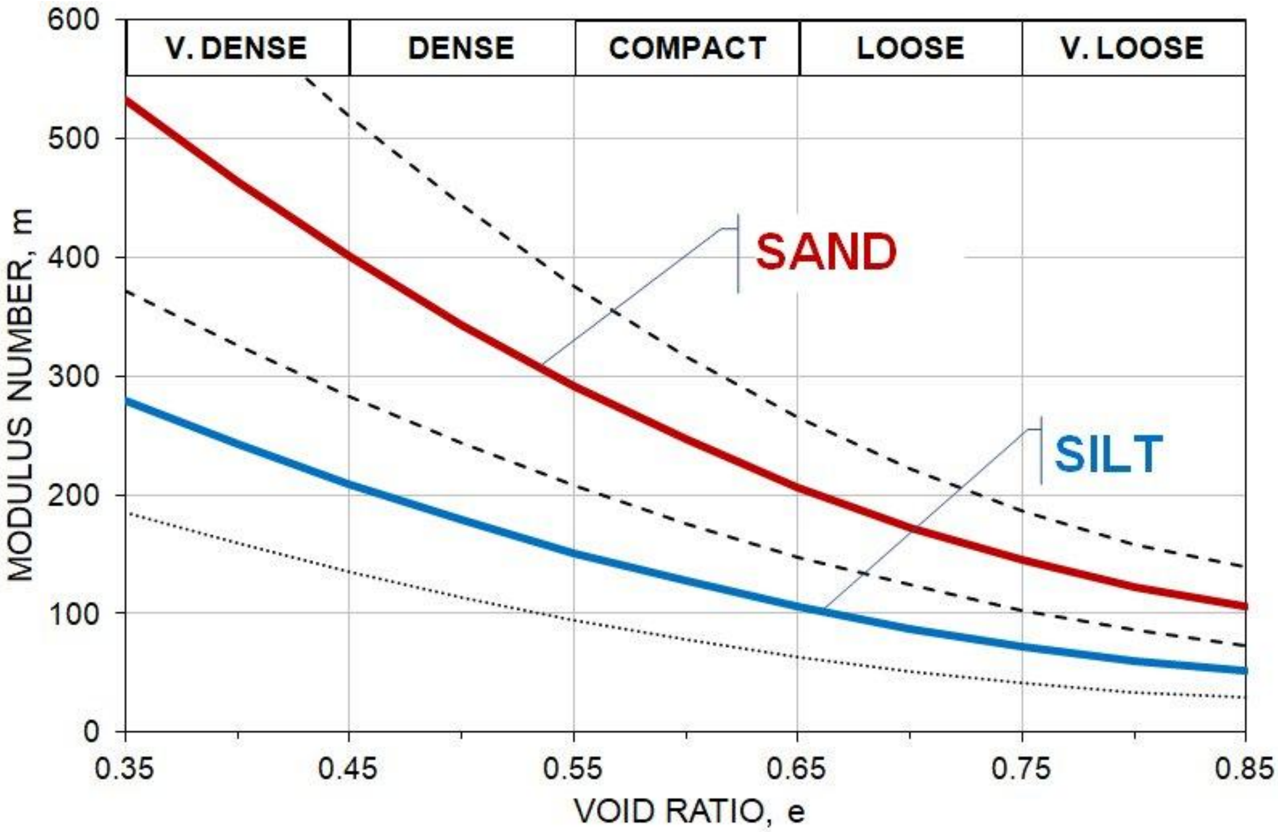

388. Massarsch, K.R. and Fellenius, B.H. 2018. In-situ tests for settlement design of compacted sand. Proceedings of the Institution of Civil Engineers, ICE, Proceedings of the Institution of Civil Engineers, (ICE) – Geotechnical Engineering 172(3): 207–217.

Cite this article
Massarsch KR and Fellenius BH (2019)
In situ tests for settlement design of compacted sand.
Proceedings of the Institution of Civil Engineers – Geotechnical Engineering **172(3)**: 207–217,
https://doi.org/10.1680/jgeen.18.00046

Research Article
Paper 1800046
Received 12/03/2018;
Accepted 24/09/2018;
Published online 31/10/2018

Keywords: design methods & aids/shallow foundations/site investigation

Geotechnical Engineering

ice Publishing

In situ tests for settlement design of compacted sand

1 Karl Rainer Massarsch DrTech, EurIng
Consulting Engineer, Geo Risk & Vibration Scandinavia AB, Stockholm, Sweden (corresponding author: rainer.massarsch@georisk.se) (Orcid:0000-0001-8906-7452)

2 Bengt H. Fellenius DrTech, PEng
Consulting Engineer, Sidney, BC, Canada (bengt@fellenius.net)

The reliability of settlement analyses depends on the selection of realistic input parameters. The tangent modulus method is proposed for the settlement analysis of uncompacted and compacted sand. The tangent modulus can be estimated based on the appropriate modulus number and corresponding stress exponents. Four concepts are presented, based on whether the modulus number is estimated from laboratory tests, empirical values or in situ tests (cone penetration tests and flat dilatometer tests). From the increase in horizontal stress following compaction, it is possible to estimate the overconsolidation ratio. A case history is presented to illustrate application of the proposed concepts.

Notation

A	net area ratio
a	empirical modulus modifier
C_M	cone stress adjustment factor
E_D	dilatometer modulus
e	void ratio
f_s	sleeve resistance
f_{s0}	sleeve resistance before compaction
f_{s1}	sleeve resistance after compaction
I_{DM}	material index
j	stress exponent
j_u	stress exponent unloading
K_0, K_{00}	at-rest earth stress coefficient for normally consolidated sand
K_1, K_{01}	at-rest earth stress coefficient (overconsolidated)
K_D	horizontal stress index
K_{D1}, $K_{D,bef}$	horizontal stress index before compaction
K_{D2}, $K_{D,aft}$	horizontal stress index after compaction
M	vertical, drained, constrained modulus
M_t	tangent modulus
m	modulus number
m_r	re-loading modulus number
m_u	modulus number for unloading
p_0	pressure applied at start of flat dilatometer test (DMT) expansion
p_1	pressure applied at end of DMT expansion
q_c	cone stress
q_{cM}	stress-adjusted cone stress
q_t	cone stress adjusted for pore water pressure on the cone shoulder
R_M	correction factor based on empirical data
u	pore water pressure
u_0	hydrostatic pore water pressure
β	exponent determined from laboratory tests
ε	strain
σ_r	reference stress = 100 kPa
σ'_m	mean effective stress
σ'_v	vertical effective stress
σ'_{v0}	vertical effective stress prior to loading
σ'_{v1}	vertical effective stress after loading
ϕ'	effective friction angle
ϕ'_0	friction angle before compaction
ϕ'_1	friction angle after compaction

1. Introduction

Deep compaction is often performed with the aim of reducing total and differential settlement and to remedy liquefaction susceptibility. With regard to settlement, however, the geotechnical literature lacks practice-oriented guidance on how to calculate settlement in sand for conditions prior to and after compaction in order to determine whether compaction is necessary and, if so, to what degree. Moreover, compaction requirements are frequently stated in ambiguous terms, such as requiring a minimum density index (formerly termed 'relative density'), which cannot be directly correlated to settlement or applied to a settlement analysis. The density index is sometimes derived from results of an in situ test – for example, a cone penetration test (CPT) or a standard penetration test – and several correlations exist for converting penetration resistance to density index. However, such correlations are limited to provide qualitative results, such as 'loose, very dense' and so on, and should not be used as input to calculations.

The main challenge in calculating the settlement of compacted sand lies in the selection of relevant input soil parameters, such as compressibility (stiffness) and stress conditions (preconsolidation stress and overconsolidation ratio (OCR)). The inability of designers to assess sand settlement reliably can be considered one of the principal limitations of the cost-efficient application of deep compaction of granular soils.

It is important to appreciate that acceptance limits for total and differential settlement can be satisfied by a multitude of compaction specifications. Thus, prescribing compaction requirements in terms of a single condition (e.g. density index, minimum penetration resistance, etc.) should be avoided as they may not be suitable and may even increase project costs. In addition, such requirements restrict the options of a specialist foundation contractor to apply innovative, more economical compaction solutions (equipment and processes).

This paper addresses how settlement can be analysed by the tangent modulus method, which – in the authors' opinion – is a transparent concept and the most suitable method for calculating the settlement of uncompacted and compacted granular soils such as silt, sand and gravel. The analysis makes use of input data based on CPT or flat dilatometer test (DMT) records.

2. The tangent modulus method

Reliable methods for characterising the compressibility of soils have been available for a long time, such as the generally applicable tangent modulus method first proposed by Ohde (1951) and Janbu (1963) for settlement analyses. The method is described in the second and third editions of the *Canadian Foundation Engineering Manual* (CGS 1985, 1992) but, regrettably, is not included in the fourth edition (CGS, 2006). Unfortunately, the method is not widely known and is therefore rarely applied in practice.

The tangent modulus is the ratio between a change of stress and the change of strain induced by that stress change, as defined by

$$1. \qquad M_t = \frac{\Delta\sigma}{\Delta\varepsilon} = m\,\sigma_r \left(\frac{\sigma'_v}{\sigma_r}\right)^{(1-j)}$$

where M_t is the tangent modulus, $\Delta\sigma$ is the change of stress, $\Delta\varepsilon$ is the change of strain, m is the modulus number (dimensionless), σ_r is the reference stress (equal to 100 kPa), σ'_v is the vertical effective stress and j is the stress exponent.

Integrating Equation 1 yields the following general relationship for determining the strain, ε, of a soil layer resulting from an increase of stress.

$$2. \qquad \varepsilon = \frac{1}{mj}\left[\left(\frac{\sigma'_{v1}}{\sigma_r}\right)^j - \left(\frac{\sigma'_{v0}}{\sigma_r}\right)^j\right]$$

Here, σ'_{v0} is the vertical effective stress prior to loading and σ'_{v1} is the vertical effective stress after loading.

The most important aspect of the tangent modulus concept is the selection of realistic input parameters, namely the stress exponent j and the virgin and re-loading modulus numbers m and m_r, respectively. The re-loading modulus number m_r applies to a stress increase in the overconsolidated condition. An important consideration when analysing settlement in sand fill is that, prior to compaction, it can be assumed that the untreated soil deposit is normally consolidated. This assumption, which is conservative, simplifies the understanding of how soil properties and stress conditions change due to vibratory compaction.

3. Stress exponent

The stress exponent j in Equation 1 defines the shape (curvature) of the load–compression relation and is based on soil type and stress conditions, which are relatively easy to estimate. For dense sand and gravel or glacial tills (overconsolidated soils), the stress exponent is usually 1·0, which indicates a linear response (elastic) to load. For loose silt and sand, j is typically 0·5, but decreases with decreasing grain size. Although j goes towards a value of 0·25 in silty soils, in practice it is usually satisfactory to assume $j = 0{\cdot}5$ (n.b. for normally consolidated condition).

3.1 Uncompacted, loose to medium dense sand: $j = 0{\cdot}5$

Uncompacted loose and medium dense (compact) sand can be assumed to be normally consolidated, with a stress exponent $j = 0{\cdot}5$. Substituting $j = 0{\cdot}5$ into Equation 2 yields Equation 3. Note that Equation 3 requires the stress to be input in units of kPa, as kPa was chosen as the unit of the reference stress σ_r (100 kPa).

$$3. \qquad \varepsilon = \frac{1}{5\,m}\left[(\sigma'_{v1})^{0{\cdot}5} - (\sigma'_{v0})^{0{\cdot}5}\right]$$

It is important to appreciate that the deformation modulus of sandy soils according to Equation 3 is non-linear.

3.2 Dense sand: $j = 1$

In dense (compacted) and very dense sand, the stress exponent $j = 1$, which means that the soil response is essentially linear-elastic. Then, inserting $j = 1$ and $\sigma_r = 100$ kPa in Equation 2 yields Equation 4 (noting, again, that the stress input must be in kPa because the reference stress is in kPa).

$$4. \qquad \varepsilon = \frac{1}{100\,m}(\sigma'_{v1} - \sigma'_{v0}) = \frac{1}{100\,m}\Delta\sigma'$$

Table 1. Typical stress exponent and modulus numbers for first loading (m) and unloading test (m_u), from Ohde (1951)

Sand type	d_{50}/d_{10}	d_{10}	e	m	j	m_u	j_u	m_u/m
Medium dense, rounded	1·35	0·20	0·64	300	0·30	900	0·30	3·00
	1·90	0·50	0·60	750	0·45	1100	0·45	1·47
Medium dense, angular	1·45	0·20	0·84	200	0·30	800	0·30	4·00
	1·90	0·50	0·68	300	0·45	1000	0·45	3·33
Gravelly	1·90	0·20	0·56	150	0·30	800	0·30	5·33
	5·00	0·65	0·46	300	0·45	1000	0·45	3·33
Fine	1·45	0·10	0·77	150	0·30	650	0·35	4·33
	1·85	0·20	0·72	250	0·40	700	0·40	2·80
	2·50	0·10	0·69	100	0·20	750	0·40	7·50
	3·50	0·20	0·56	200	0·30	800	0·40	4·00

4. Modulus number

The most challenging aspect of estimating settlement is the selection of realistic values of soil stiffness (compressibility), that is, the modulus number m. Four approaches can be employed to determine the modulus number of coarse-grained (sandy and gravelly) soils

(*a*) laboratory tests on reconstituted samples
(*b*) experience-based (empirical) values
(*c*) relationships derived from CPT soundings
(*d*) relationships derived from DMT records.

These four alternative procedures are now discussed in turn.

4.1 Modulus number from laboratory tests

As it is generally difficult to perform laboratory tests on undisturbed sand samples, the selection of the modulus number must be determined from reconstituted soil samples. Table 1 shows the relationship between different geotechnical parameters (soil type, d_{50}/d_{10}, d_{10} and void ratio e) from laboratory tests, as presented by Ohde (1951). Note that Ohde (1951) used the ratio d_{50}/d_{10}, which is slightly different to C_u (d_{60}/d_{10}). The test data reported by Ohde (1951) serve as guidance only, as the modulus numbers in loading (first loading, virgin condition) were back-calculated for different values of j.

The modulus numbers and stress exponents were also determined for the case of unloading and showed values about 1·5–7·5 times larger than the modulus numbers back-calculated for virgin loading.

4.2 Modulus number from empirical data

Based on data presented by Janbu (1963), values for m and j according to soil type of coarse-grained soil were published in the *Canadian Foundation Engineering Manual* (CGS, 1985, 1992). Table 2 shows the typical range and average value of m for two cases of the stress exponent (0·5 and 1·0) as relevant to soil type.

In more recent work, Janbu (1985) updated typical values of the modulus number m for normally consolidated silt and sand ($j = 0·5$), categories that are of interest with regard to settlement analyses for soil compaction. Janbu (1985) presented the modulus number as a function of porosity n, herein converted to the more widely used void ratio e. Figure 1 shows

Table 2. Typical stress exponent and modulus numbers for granular soils (CGS, 1992)

Soil type	Stress exponent, j	Range of m	Average m
Till, very dense to dense	1	1000–300	650
Gravel	1	400–40	220
Sand			
Dense	1	400–250	325
Compact	1	250–150	200
Loose	0·5	150–100	125
Silt			
Dense	1	200–80	140
Compact	1	80–60	70
Loose	0·5	60–40	50

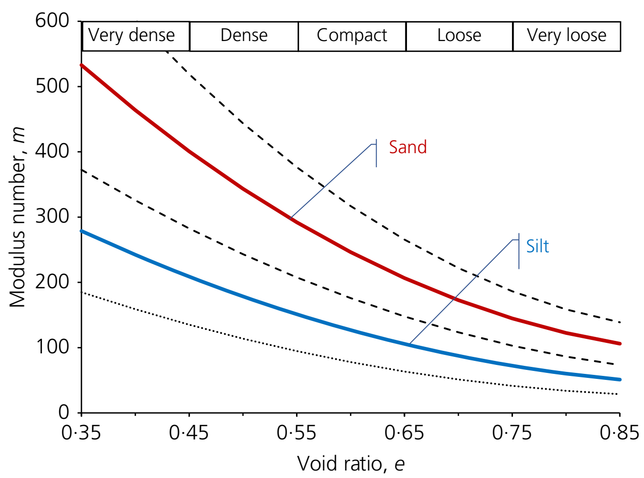

Figure 1. Typical modulus numbers for normally consolidated sand and silt with upper and lower boundaries for silt (lower heavy line) and sand (upper heavy line) (cf. Table 2). Data derived from Janbu (1985)

the modulus number derived by Janbu (1985) as a function of void ratio for silt and sand and different degrees of density. Also indicated in the figure is the approximate range (and lower/upper boundaries) of modulus numbers for the respective soil category (sand and silt) according to the classification used in Table 2.

The range of modulus numbers shown in Figure 1 is considered the most reliable, empirical database for soil compaction projects and is in good agreement with previously published empirical data (cf. Table 2).

4.3 Modulus number from CPTs

The CPT – and variations thereof, such as the CPTU (with pore water pressure measurement) or the SCPT (with seismic downhole test) – is today the most widely used field investigation method on sand compaction projects. The CPT is standardised, thereby reducing the risk that equipment and operation affect the measured parameters (ISO, 2012; ISSMGE, 1999). Other field investigation methods are heavy dynamic probing, the standard penetration test or the DMT. The CPT has the advantage of generating geotechnical information at low cost in most soils suitable for compaction. An additional important advantage of the CPTU is that it measures three independent parameters – the cone stress q_c, sleeve resistance f_s and pore water pressure u. As will be demonstrated in this paper, the benefit of repeatable and accurate measurements of q_c and f_s is the key to the successful design of sand compaction projects, particularly in hydraulic fills. The measured cone stress q_c is usually corrected for pore water pressure u using

5. $$q_t = q_c + u(1 - A)$$

where q_t is the cone stress adjusted for pore water pressure on the cone shoulder and A is the net area ratio.

In sandy soils, the pore water pressure will be low compared with the measured cone stress; thus, it can be assumed that $q_c \approx q_t$. Therefore, q_c will be used subsequently.

The method relies on knowledge of the mean effective stress, expressed by

6. $$\sigma'_m = \sigma'_v \left(\frac{1 + 2K_0}{3}\right)$$

where σ'_m is the mean effective stress, σ'_v is the vertical effective stress and K_0 is the at-rest earth stress coefficient.

For uncompacted (normally consolidated) sand, K_0 can be estimated from the simplified relationship proposed by Jáky (1948)

7. $$K_0 \sim 1 - \sin(\phi')$$

where ϕ' is the effective friction angle. A typical value of K_0 for uncompacted sand ($\phi' \approx 33°$) would be 0·43. In overconsolidated granular soils, it is necessary to estimate the effective horizontal stress based on empirical correlations or engineering judgement. The cone stress is influenced by depth and, thus, by the effective confining stress. Massarsch (1994) proposed a stress adjustment factor (C_M) to take into account the effect of mean effective stress σ'_m on the cone stress measured in sandy soils. This stress adjustment factor is given by

8. $$C_M = \left(\frac{\sigma_r}{\sigma'_m}\right)^{0 \cdot 5}$$

where $C_M \leq 2 \cdot 5$, σ_r is the reference stress (= 100 kPa) and σ'_m is the mean effective stress.

The stress-adjusted cone stress q_{cM} can now be calculated as

9. $$q_{cM} = q_c C_M = q_c \left(\frac{\sigma_r}{\sigma'_m}\right)^{0 \cdot 5}$$

The stress-adjusted cone stress q_{cM} is independent of depth, which is very useful for geotechnical design of uncompacted and compacted fill, as will be demonstrated later. Massarsch (1994) proposed correlating the modulus number for granular soils with the stress-adjusted cone stress using

10. $$m = a\left(\frac{q_{cM}}{\sigma_r}\right)^{0 \cdot 5}$$

where a is an empirical modulus modifier. The modulus modifier a reflects soil type and varies within a relatively narrow range for each soil category, as indicated in Table 3.

Substituting Equations 6 and 9 into Equation 10 yields

11. $$m = a\left[\frac{q_c}{(\sigma_r\,\sigma'_v)^{0 \cdot 5}}\left(\frac{3}{1 + 2K_0}\right)^{0 \cdot 5}\right]^{0 \cdot 5}$$

Table 3. Modulus modifier, a, for different soil types (Massarsch and Fellenius, 2014)

Soil type	Modulus modifier, a
Silt, organic soft	7
Silt, loose	12
Silt, compact	15
Silt, dense	20
Sand, silty loose	20
Sand, loose	22
Sand, compact	28
Sand, dense	35
Gravel, loose	35
Gravel, compact	40
Gravel, dense	45

For normally consolidated soil with K_0 ranging between 0·4 and 0·6, the numerical value of the term $[3/(1+2K_0)]^{0.5}$ is between 1·29 and 1·17. That term can be approximated by setting it equal to the mean of the range (i.e. 1·23), resulting in the following simple relationship for estimating the modulus number in loose and medium dense (compact) sand.

12. $$m = a\left[q_c \frac{1.23}{(\sigma_r \sigma_v')^{0.5}}\right]^{0.5}$$

In heavily compacted sand, K_0 can increase beyond 1 and, in that case, Equation 10 should be used rather than Equation 12.

An important advantage of determining the modulus number from Equations 10, 11 or 12 is that CPT data are normally available for compaction projects, both prior to and after compaction. The cone stress reflects the variation of soil stiffness with depth more accurately than simple classification concepts based on soil type (cf. Table 3) and the improvement effect of compaction is reflected more realistically by the increase in stress-adjusted cone stress q_{cM} (Equation 9) than by the unadjusted q_c. The change in stress-adjusted cone resistance due to compaction can be considered a more reliable indicator of the increase in soil modulus than selection of empirical values.

As a result of soil compaction, sand becomes generally overconsolidated and deformation properties change from the normally consolidated state ($j = 0.5$) to the overconsolidated state ($j = 1$). The modulus number for the portion of the applied stress that lies in the overconsolidated stress range is much larger than that in the virgin range.

Ohde (1951) presented values of the modulus number for sand in virgin loading and in unloading, and indicated that the ratio m_u/m varied between about 2 to 6. The ratio m_u/m can be taken as a lower boundary value of the ratio m_r/m between the re-loading modulus m_r and the virgin modulus m.

4.4 Modulus number from DMT records

Another in situ test suitable for evaluation of soil compaction projects is the DMT, which is a relatively recent field testing addition. Guidelines for DMT equipment and application techniques have been issued by ISSMGE Technical Committee 16 (Marchetti *et al.*, 2001). For a detailed description of the DMT, recent developments in data interpretation and practical application of results, readers are referred to the geotechnical literature (e.g. the proceedings of the third DMT conference (Marchetti, 2015)).

The test procedure involves advancing the dilatometer blade into the ground. Readings are taken at depth intervals of 200 mm by inflating a membrane and taking pressure readings. These 'raw' pressure readings are corrected and subsequently converted into two pressure values, p_0 and p_1. A key characteristic that distinguishes the DMT from other in situ methods is its ability to measure parameters that reflect the stress conditions in the horizontal direction. From the derived p_0 and p_1 values, the following DMT index parameters are calculated.

13. $$I_{DM} = \frac{p_1 - p_0}{p_0 - u_0}$$

14. $$K_D = \frac{p_0 - u_0}{\sigma_{v0}'}$$

15. $$E_D = 34.7(p_1 - p_0)$$

Here, I_{DM} is the material index (nomenclature modified in order to avoid confusion with the density index I_D), K_D is the horizontal stress index, E_D is the dilatometer modulus, u_0 (equal to p_1) is the hydrostatic pore water pressure, σ_{v0}' is the vertical effective stress, p_0 is the pressure applied at start of expansion and p_1 is the pressure applied at end of expansion.

Marchetti *et al.* (2001) suggested that a vertical, drained, constrained modulus (M) can be estimated from the dilatometer modulus E_D using

16. $$M = R_M E_D$$

where R_M is a correction factor based on empirical data (Marchetti, 1980) and

- $I_{DM} < 0.6$, $R_M = 0.14 + 2.36\log K_D$
- $I_{DM} > 3$, $R_M = 0.5 + 2\log K_D$
- $0.6 < I_{DM} < 3$, $R_M = R_{M,0} + (2.5 - R_{M,0})\log K_D$, with $R_{M,0} = 0.14 + 0.15(I_{DM} - 0.6)$.

If $K_D > 10$, $R_M = 0.32 + 2.18\log K_D$; if $R_M < 0.85$, assume $R_M = 0.85$. The modulus number m can then be estimated according to Equation 1. By rearranging terms, the following relationship is obtained.

17. $$m = \frac{M_t}{\sigma_r}\left(\frac{\sigma_v'}{\sigma_r}\right)^{j-1}$$

In the case of normally consolidated sand, assuming $j = 0.5$, the modulus number m is obtained from

18. $$m = M_t\left(\frac{1}{\sigma_r \sigma_v'}\right)^{0.5}$$

For the case of compacted sand, assuming $j = 1$, the following simple relationship is obtained.

$$m = \frac{M_t}{\sigma_r} \tag{19}$$

Thus, the modulus number m can be determined by dividing M (in kPa) by 100 (the reference stress, $\sigma_r = 100$ kPa).

5. Preconsolidation

A critical aspect in every settlement analysis is the determination of the preconsolidation stress and the OCR. Compaction of soil layers increases horizontal stresses through the application of a large number of hysteretic loading cycles (Duncan and Seed, 1986; Massarsch, 2002). The compacted sand becomes overconsolidated, which means that the horizontal effective stress also increases. This effect is reflected in the CPT as an increased sleeve resistance f_s. A similar effect is measured by the DMT as an increase in the horizontal stress index K_D. In the following, two methods for estimating the increase in the OCR due to compaction based on in situ tests are described.

5.1 CPT – sleeve resistance

Although some uncertainty exists regarding the accuracy of sleeve resistance measurements, the ratio of sleeve resistance after and before compaction can be considered a reliable indicator of horizontal stress change (Massarsch and Fellenius, 2002; Robertson, 2016). The sleeve resistance depends on the at-rest earth stress coefficient K_0, the vertical effective stress σ'_v and the friction angle ϕ'. As the effective overburden stress is essentially unchanged by the compaction, the ratio between the sleeve resistance after and before compaction, f_{s1}/f_{s0}, is an indication of the change in horizontal earth stress, assuming free-draining conditions. The ratio of the earth stress after and before compaction, K_{01}/K_{00}, can be estimated from

$$\frac{K_{01}}{K_{00}} = \frac{f_{s1}}{f_{s0}} \frac{\tan(\phi'_0)}{\tan(\phi'_1)} \tag{20}$$

where K_{00} is the at-rest earth stress coefficient before compaction, K_{01} is the at-rest earth stress coefficient after compaction, f_{s0} is the sleeve resistance before compaction, f_{s1} is the sleeve resistance after compaction, ϕ'_0 is the friction angle before compaction and ϕ'_1 is the friction angle after compaction.

The friction angle increases after compaction, typically by about 5°, which corresponds to a friction angle ratio ($\tan(\phi'_0)/\tan(\phi'_1)$) of about 0·85. Thus, Equation 20 can be modified as follows

$$\frac{K_{01}}{K_{00}} = 0{\cdot}85\,\frac{f_{s1}}{f_{s0}} \tag{21}$$

5.2 DMT – horizontal stress index

The dilatometer measures two values of horizontal stress (see Equation 14) from which the horizontal stress index K_D is calculated. It is difficult at present to estimate the coefficient of earth stress K_0 directly from DMT measurements, especially in granular soils, as stress history plays an important role. However, the ratio of the earth stress after and before compaction, K_{01}/K_{00}, can be estimated from the ratio of the horizontal stress index after compaction (K_{D1}) and before compaction (K_{D0}).

$$\frac{K_{01}}{K_{00}} = \frac{K_{D1}}{K_{D0}} \tag{22}$$

The DMT can be expected to give more reliable information regarding the changes in horizontal earth stress than the CPT sleeve resistance, as it actually measures stress changes in the horizontal direction.

5.3 Estimation of OCR

In natural sand deposits, it can be difficult to determine reliably whether or not a soil is overconsolidated, and if so, by how much. However, as mentioned earlier, in the case of sand fill, the stress conditions prior to compaction can be assumed to be normally consolidated (a conservative assumption). The increase in sleeve resistance then provides information regarding the increase in horizontal stress (Equation 21). Relationships between the increase in horizontal effective stress and the OCR have been proposed in the geotechnical literature (e.g. Massarsch and Fellenius, 2014).

$$\frac{K_1}{K_0} = \mathrm{OCR}^{\beta} \tag{23}$$

Rearranging of terms yields

$$\mathrm{OCR} = \left(\frac{K_1}{K_0}\right)^{1/\beta} \tag{24}$$

where K_0 is the at-rest earth stress coefficient for normally consolidated sand, K_1 is the at-rest earth stress coefficient for overconsolidated (compacted) sand and β is an exponent determined from laboratory tests.

Based on calibration chamber tests, Schmertmann (1975) recommended $\beta = 0{\cdot}42$ and Lunne and Christophersen (1983) suggested $\beta = 0{\cdot}45$. Jamiolkowski *et al.* (1988) proposed a range from 0·38 to 0·44 for medium dense sand. Equation 24 implies that a relatively small increase in the earth stress ratio K_1/K_0, say by a factor of 2, results in a significant increase in the OCR, ranging from 4 to 7 (depending on the value of β).

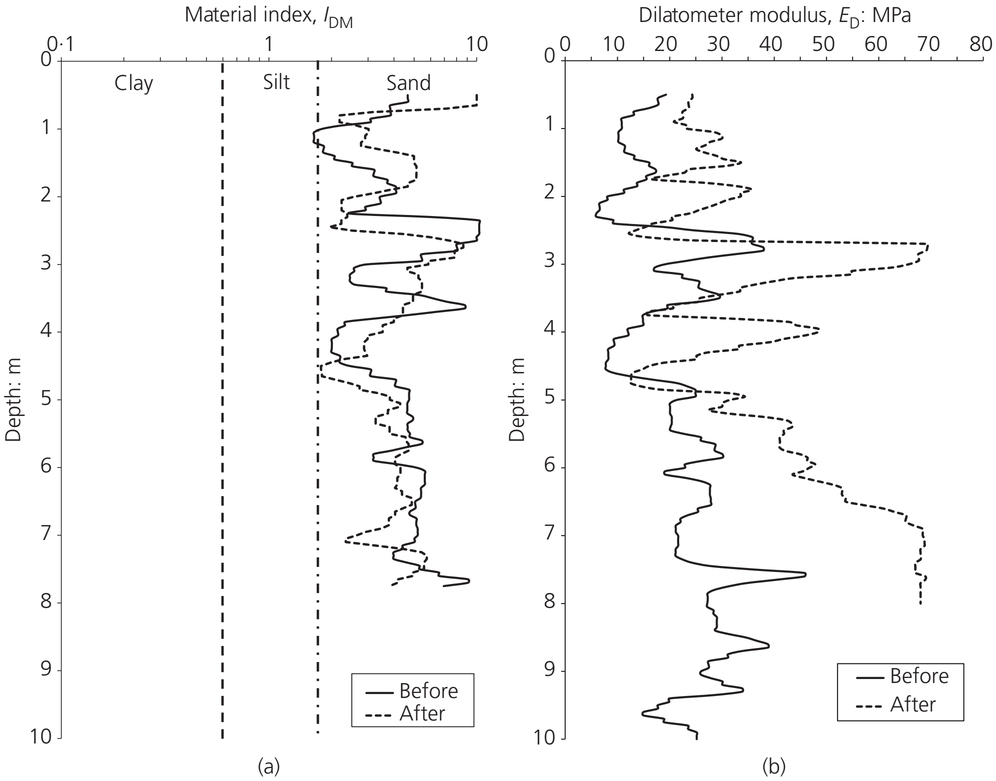

Figure 2. (a) Material index and (b) dilatometer modulus before and after compaction, interpreted from Marchetti (1980)

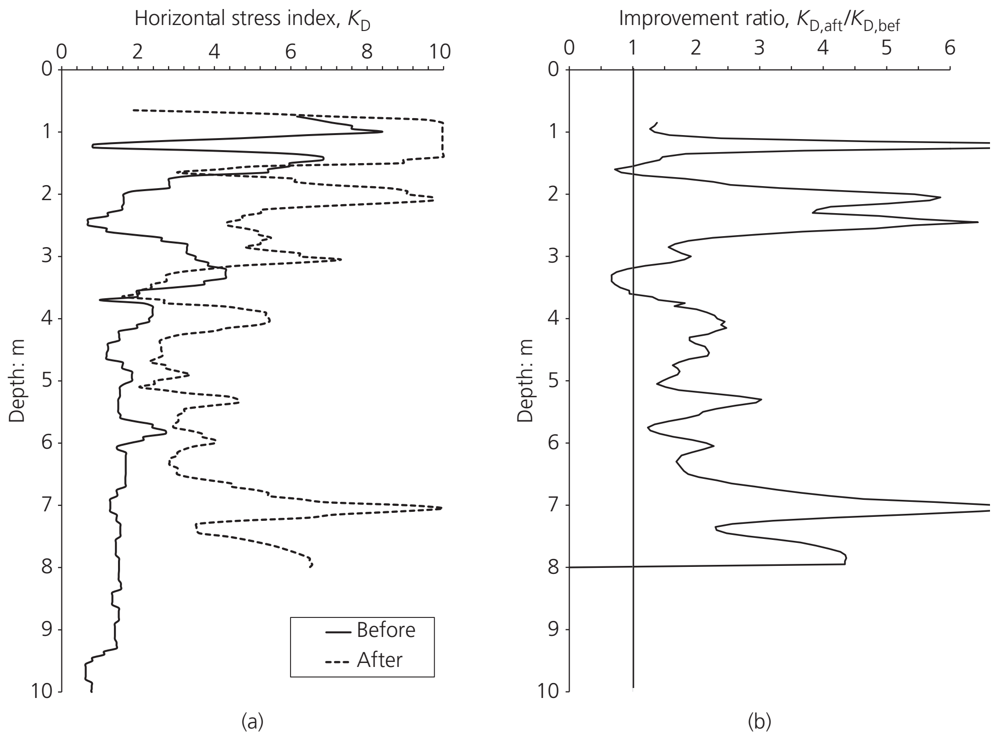

Figure 3. (a) Horizontal stress index before and after compaction, and (b) increase in horizontal stress ($K_{D,aft}/K_{D,bef}$) due to compaction

The DMT calibration chamber test data reported by Lee *et al.* (2011) were re-analysed and a close relationship between the change in horizontal stress index K_D and OCR was found. Increase in horizontal stress can be estimated from Equation 25 using the normalised horizontal stress index $K_{D,aft}/K_{D,bef}$ and the OCR.

$$\text{25.} \qquad \text{OCR} = \left(\frac{K_{D,\,aft}}{K_{D,\,bef}}\right)^{2\cdot1}$$

According to the DMT data reported by Lee *et al.* (2011), the exponent 2·1 ($1/\beta$) corresponds to a β-exponent of 0·48 (see Equation 24). Thus, the empirically determined β-exponent obtained from triaxial tests appears to be just slightly lower than the equivalent exponent derived from compression chamber tests using DMT data.

6. Case history: Damman, Saudi Arabia

Marchetti (1980) reported early DMT data from a compaction project in Damman, Saudi Arabia. The case history illustrates how the constrained modulus and the modulus number can be assessed by two different methods (CPT and DMT).

A sand fill was compacted by vibroflotation from the ground surface down to approximately 13 m depth. Compaction treatment was carried out at a triangular grid at 2 m spacing. Only limited information is provided about the compaction procedure. The soil type was medium–fine loose sand (hydraulic fill) including some silty pockets. The groundwater level was assumed at 4 m depth. CPTs were performed prior to compaction only; however, no actual CPT measurements were reported, only that in the upper few metres depth, the cone stress q_c ranged from 1 MPa to 5 MPa and was approximately constant ($q_c \approx 5$ MPa) below that depth.

DMT measurements were carried out both prior to and after compaction (at some distance from the CPTs). The DMT results, as reported by Marchetti (1980), were digitised and interpreted with the objective of determining the tangent modulus numbers according to the procedure outlined above. DMT measurements were also performed several weeks after compaction (no exact time given) at the centroid of the vibroflotation grid points. Measurements after compaction had to stop at about 8 m depth as the capacity of the pushing rig was reached. Figure 2 shows the material index I_{DM} and the dilatometer modulus E_D before and after compaction. The data were digitised from the original reference (Marchetti, 1980) and re-interpreted.

The material index before and after compaction indicated that the fill material consisted of sand with occasional silt layers. The dilatometer modulus before compaction increased from about 10 MPa at 2 m depth to about 35 MPa at 8 m depth. After compaction, E_D had increased to between 15 and 65 MPa. Measurements after compaction were only taken to 8 m depth.

Figure 3 shows the horizontal stress index K_D before and after compaction, as well as the horizontal stress ratio $K_{D,aft}/K_{D,bef}$. In the compacted zone, down to 8 m depth, K_D increased due to treatment. This is more clearly shown by the horizontal stress ratio $K_{D,aft}/K_{D,bef}$, which indicates an increase of between 1·5 and 3·0 (with some exceptionally high values close to the ground surface). Obviously, the soil compaction resulted in a marked increase in the horizontal effective stress.

From the change in horizontal stress ($K_{D,aft}/K_{D,bef}$) it is possible to estimate, based on Equation 25, the OCR (assumed stress exponent of 2·1). The so-determined OCR is shown in Figure 4.

Due to the variability of soil layers, the OCR also fluctuated. However, the average OCR after compaction can be estimated to range between 2 and 6. Higher OCR values can be attributed to the existence of layers of lower strength prior to compaction.

The constrained modulus was calculated from the DMT measurements according to the concept outlined above using Equation 16. Subsequently, the modulus number m was calculated for the sand prior to and after compaction, according to

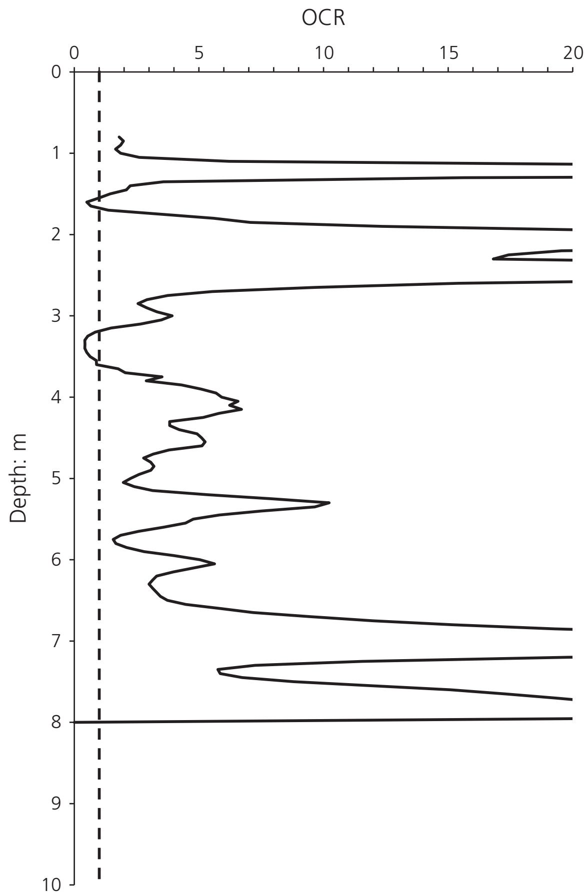

Figure 4. OCR determined from Figure 3, based on Equation 25

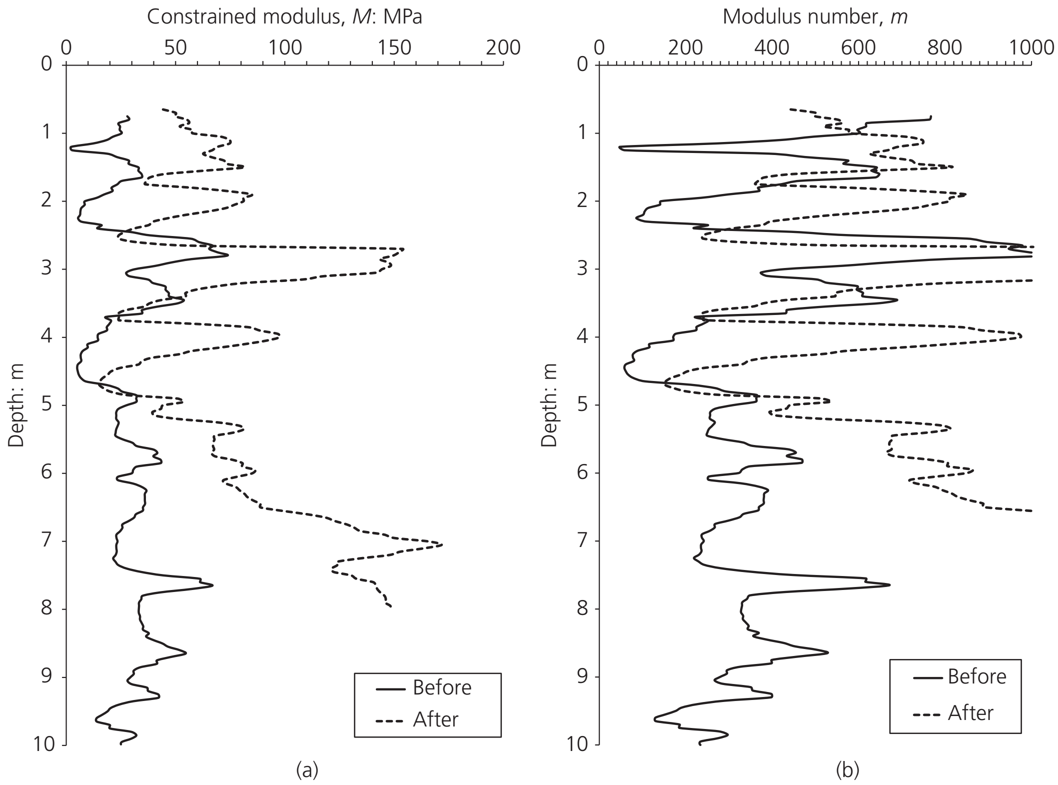

Figure 5. (a) Constrained modulus determined from dilatometer modulus E_D (cf. Figure 3) and (b) modulus number

Equation 17, assuming $j = 0{\cdot}5$ for loose sand and $j = 1$ for dense sand. The constrained modulus and the modulus number before and after compaction are shown in Figure 5.

The constrained modulus and the modulus number varied significantly with depth both prior to and after treatment. The modulus number before compaction (ranging between 100 and 400) is in reasonable agreement with the range of *m* values for loose to medium dense sand (Figure 1). However, both the constrained modulus *M* and the modulus number *m* after compaction were high, with *m* between 200 and 1000.

Although no actual CPT data were reported, an attempt was made to estimate the constrained modulus and the modulus before compaction based on the values given by Marchetti (1980) (before compaction $q_c = 5$ MPa and after compaction $q_c = 10$ MPa). The parameters listed in Table 4 were assumed in the interpretation of the q_c data.

Figure 6 shows the constrained modulus *M* and the derived modulus number *m* for the cases prior to and after compaction, according to the assumptions in Table 4.

While the *m* values from DMTs and CPTs prior to compaction were found to be in reasonable agreement, this was not the case for the post-compaction *m* values: *m* values according to the CPT data are significantly lower than those based on the DMT. The modulus numbers based on the CPT data are in close agreement with the values given in Table 2 and shown in Figure 1. The modulus number for uncompacted, loose sand varies between 100 and 150, according to Table 2, which is in reasonable agreement with Figure 6. Similarly good agreement was obtained for the case after compaction, which, according to Table 2, should be in the range 250–400. Thus, the modulus numbers obtained from CPTs were in agreement with Janbu's data than the values from DMTs, which were beyond the upper boundary of values given in Figure 1 and Table 2. However, as the DMT is affected by the increase in horizontal stress, the modulus number derived from the DMT may give a more realistic values of soil compressibitlity.

Table 4. Parameters used in analysis of *M* and *m* from CPT data

	Before compaction	After compaction
q_c: MPa	5	10
K_0	0·46	1·0
a	22	28
j	0·5	1·0

7. Conclusions

Settlement is often the primary requirement for vibratory compaction of sand. As a first step, it is important to determine

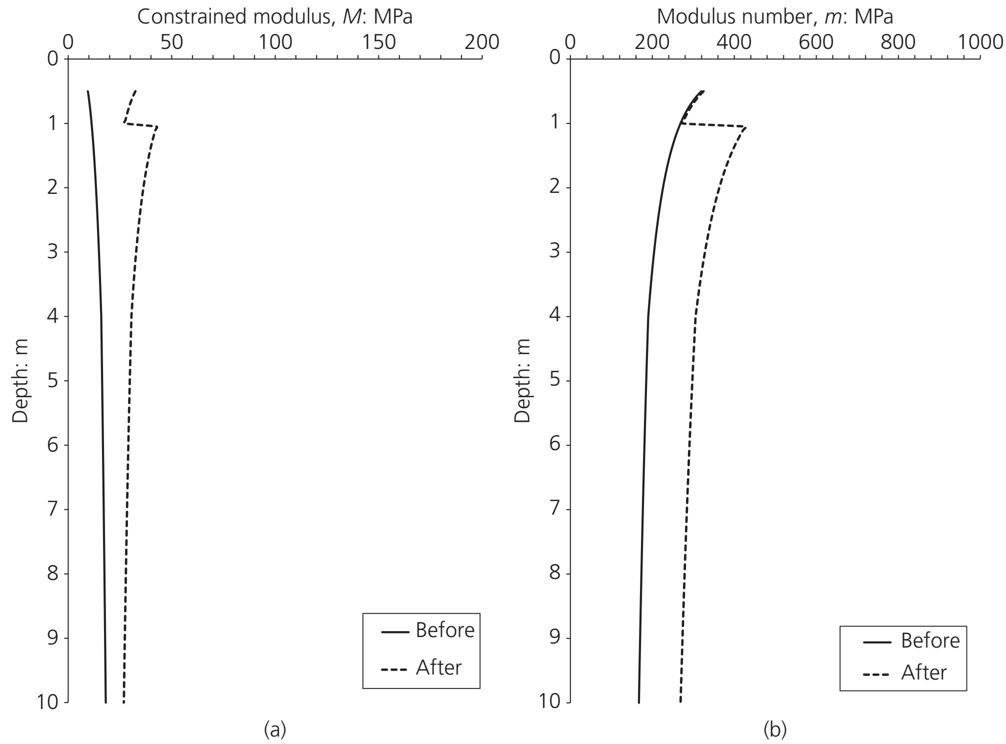

Figure 6. (a) Constrained modulus and (b) modulus number before and after compaction obtained with assumed CPT data (see Table 4). Note that the same scale was chosen as in Figure 4 to facilitate comparison of data

the settlement of untreated soil in order to establish whether – and to what degree – compaction will be required.

The tangent modulus concept is the preferred method of settlement analysis as it is a transparent approach that allows adjustment of settlement calculations to soil type, allowing the assumption of non-linear modulus and overconsolidation stress changes. The tangent modulus method can be used for settlement analyses in all soil types because it takes into account the stress dependency of the soil modulus.

A reason for the limited application of the tangent modulus method on compaction projects is the uncertainty of choosing appropriate values of the modulus number prior to and after compaction. An important input parameter for settlement analysis is the selection of the modulus number m. Recommended values of the modulus number for silt and sand were proposed by Janbu (1985) for normally consolidated soils with $j = 0{\cdot}5$ (see Figure 1). The upper range of values can also be considered relevant for compacted (overconsolidated) sand and silt.

A method for estimating the modulus number, based on the CPT, has been outlined. This method can be used for pre- and post-compaction conditions and is based on the stress-adjusted cone stress q_{cM}. Using in situ tests has the advantage of determining the soil modulus based on actual field measurements – in particular, changes in soil conditions between before and after compaction.

An alternative method of determining the constrained modulus M is based on DMT results. The constrained modulus M (which is derived from E_D) involves a correction factor (R_M) that needs to take into account the changes in effective stress K_D.

The preconsolidation effect due to compaction can be estimated based on the increase in horizontal stress (Equations 23 and 24). A stress exponent of $\beta = 0{\cdot}48$, verified by calibration chamber tests using DMT, was found to be in agreement with previously suggested values.

A case history where CPT (before compaction) and DMT (before and after compaction) data were available was presented. The concept of estimating the modulus number and the OCR was illustrated using CPT and DMT results.

The DMT data showed a marked increase in the horizontal stress index K_D. The OCR was determined based on the ratio of the horizontal stress index. The average increase in OCR varied but was, on average, between 2 and 6.

The modulus number was estimated from CPT and DMT data. The agreement between empirical values of m and the values derived from the CPT was found to be good. This approach will provide settlement estimates on the conservative side. The modulus number calculated from the DMT constrained modulus was at the upper limit of the empirical values. This is an important observation for the cases of compacted sand and silt, as the presently available data (Janbu, 1985) were obtained for normally consolidated soils. In compacted (overconsolidated) soils, the modulus number can be higher, as indicated in Table 1 by the unloading ratio m_u/m in the range 3·0–7·5. Thus, DMTs could provide more economical settlement estimates for compacted sand. However, further studies are warranted to verify that the modulus numbers determined by the DMT can be applied in practice.

Acknowledgement
The inspiration to prepare this paper came from discussions between the first author and the late Professor Silvano Marchetti.

REFERENCES

CGS (Canadian Geotechnical Society) (1985) *Canadian Foundation Engineering Manual*, 2nd edn. Canadian Geotechnical Society, Richmond, BC, Canada.

CGS (1992) *Canadian Foundation Engineering Manual*, 3rd edn. Canadian Geotechnical Society, Richmond, BC, Canada.

CGS (2006) *Canadian Foundation Engineering Manual*, 4th edn. Canadian Geotechnical Society, Richmond, BC, Canada.

Duncan JM and Seed RB (1986) Compaction-induced earth pressures under K_0 conditions. *Journal of Geotechnical Engineering ASCE* **112(1)**: 1–22.

ISO (International Organization for Standardization) (2012) 22476-1:2012: Geotechnical investigation and testing – field testing. Part 1: Electrical cone and piezocone penetration test, ISO/TC 182/SC 1. ISODIS 22476-1. ISO, Geneva, Switzerland.

ISSMGE (International Society for Soil Mechanics and Geotechnical Engineering Technical Committee 16) (1999) International reference test procedure (IRTP) for the cone penetration test (CPT) and the cone penetration test with pore pressure (CPTU). In *Proceedings of the XIIth ECSMGE* (de Cock F, Legrand C and Lehane B (eds)). Balkema, Rotterdam, the Netherlands, pp. 2195–2222.

Jáky J (1948) Pressure in silos. In *Proceedings of the 2nd ICSMFE, Rotterdam, the Netherlands,*, vol. 1, pp. 103–107.

Jamiolkowski M, Ghionna VN, Lancelotta R and Pasqualini E (1988) New correlations of penetration tests for design practice. In *Proceedings of Symposium on Penetration Testing. ISOPT-1, Orlando, FL, USA* (de Ruiter J (ed.)). Balkema, Rotterdam, the Netherlands, pp. 263–296.

Janbu N (1963) Soil compressibility as determined by oedometer and triaxial tests. In *Proceedings of III European Conference on Soil Mechanics and Foundation Engineering (ECSMF), Wiesbaden, Germany*, vol. 1, pp. 19–25; vol. 2, pp. 17–21.

Janbu N (1985) Soil models in offshore engineering. *Géotechnique* **35(3)**: 241–281, https://doi.org/10.1680/geot.1985.35.3.241.

Lee M, Choi S, Kim M and Lee W (2011) Effect of stress history on CPT and DMT results in sand. *Journal of Engineering Geology* **117**: 259–265.

Lunne T and Christophersen HP (1983) Interpretation of cone penetrometer data for offshore sands. *Proceedings of the Offshore Technology Conference, Richardson, TX, USA*, Paper no. 4464.

Marchetti S (1980) In-situ tests by flat dilatometer. *Journal of Geotechnical Engineering ASCE* **106(3)**: 299–321.

Marchetti S (2015) Some 2015 updates to the TC16 DMT report. *Proceedings of the 3rd International Conference on the Flat Dilatometer (DMT'15), Rome, Italy*, pp. 43–65.

Marchetti S, Monaco P, Totani G and Calabrese M (2001) The flat dilatometer test (DMT) in soil investigations. Report by ISSMGE Committee TC16. *Proceedings of In Situ 2001, International Conference On In situ Measurement of Soil Properties, Bali, Indonesia* (Rahardjo PP and Lunne T (eds)), pp. 95–131.

Massarsch KR (1994) Settlement analysis of compacted fill. *Proceedings of the 13th ICSMFE, New Delhi, India*, vol. 1, pp. 325–328.

Massarsch KR (2002) Effects of vibratory compaction. *Proceedings of TransVib 2002 – International Conference on Vibratory Pile Driving and Deep Soil Compaction, Louvain-la-Neuve, Belgium*, Keynote Lecture, pp. 33–42.

Massarsch KR and Fellenius BH (2002) Vibratory compaction of coarse-grained soils. *Canadian Geotechnical Journal* **39(3)**: 695–709.

Massarsch KR and Fellenius BH (2014) Use of CPT for design, monitoring, and performance verification of compaction projects. *Proceedings of the 3rd International Symposium on Cone Penetration Testing* (Robertson PK and Cabal KL (eds)). Omnipress, Madison, WI, USA, pp. 1187–1200.

Ohde J (1951) *Grundbaumechanik*. Ernst & Sohn, Berlin, Germany (in German).

Robertson PK (2016) Estimating K_0 in sandy soils using the CPT. *Proceedings of the 5th International Conference on Geotechnical and Geophysical Site Characterization (ISC'5), Gold Coast, Australia* (Lehane BM, Acosta-Martínez HE and Kelly R (eds)), pp. 515–520.

Schmertmann JH (1975) Measurement of in-situ shear strength. *Proceedings of American Society of Civil Engineers Specialty Conference on In-Situ Measurement of Soil Properties, Raleigh, NC, USA*, ASCE, Reston, VA, USA, vol. 2, pp. 57–138.